MULTI-TONE ADAPTIVE VIBRATION ISOLATION OF ENGINEERING STRUCTURES

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Abstracts

We present a multi-tone narrow band tracking control algorithm to isolate periodic disturbances from engineering structures. Conditions for fast convergence of control parameters are discussed. It is shown that the choice of appropriate tap delay time and number of taps is important in achieving good convergence. We implemented the algorithm in a digitat computer to attenuate a two-tone periodic disturbance from a flexible aluminum truss structure. Experimental results showed that disturbance rejection of over 25 dB was achieved.

Introduction

Machinery vibration is an important issue in many engineering applications [1], including space structures, aircraft, dams and commercial buildings. In space structures, high speed reaction wheels for on board attitude control systems and cryo-coolers for on board imaging instruments can generate unacceptable levels of vibration, Similarly, engines in aircraft, large generators in dams, large transformers in power stations, and escalators or ventilation fans in commercial buildings can also generate undesirable levels of vibration in these structures, While these disturbances are periodic in nature, their frequency, magnitude, and phase may vary slowly with time. The propagation of such disturbances, being magnified by the resortant dynamics, can interfere with the normal operation of sensitive instruments present in the structure. Also this can cause fatigue damage to the structure or can be very annoying to its occupants

One of the most popular methods to counter the problem is to place a passive mount (passive isolation) between the disturbance source and the structure. A passive mount is essentially a combination of a soft spring and a damper that can significantly reduce disturbance propagation from the source. While a soft passive mount may be necessary to attenuate a broad band disturbance, it introduces a soft connection that may not be acceptable from structural integrity considerations.

Many investigators including those in reference [1-5] have proposed to augment passive mounts with closed loop feedback systems i.e., active isolation to improve total isolation performance. Such broad band isolation suffers from the same problem as passive isolation, Broad band feedback control essentially further softens [6] the existing passive mount and in-turn lowers the comer frequency to introduce additional isolation performance, Also, in this control scheme it is difficult to obtain good performance over a large frequency **band,** since such control scheme requires a gradual roll off for stability.

Investigators including those in reference [4,5] presented narrow band tracking type feedback control methods, variants of the Least Mean Square-s (LMS) algorithm [10,12], for isolation of periodic disturbances by softening the passive mount selectively only at the disturbance frequencies. In this method, the general softening of the mount was avoided and structural integrity was not compromised Performance over 40 dB was achieved at tbc targeted frequencies, The method presented in reference [4] uses phase difference networks [7,8] to accommodate very large variations in the frequency of each harmonic. However, in both schemes, each

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disturbance harmonic has to be dealt with separately. This makes the schemes very cumbersome and computationally more demanding *when the* disturbance consists of more than one harmonic, In this paper, wc discuss a method for implementing the LMS algorithm to isolate a disturbance consisting of multiple harmonics whose frequencies (amplitudes and phase) may vary over large ranges. Wc discuss the convergence properties of the method and present a numerical scheme to maximize the convergence rate by choosing appropriate number of taps (control weigh(s) and tap delay time (sequential sampling time).

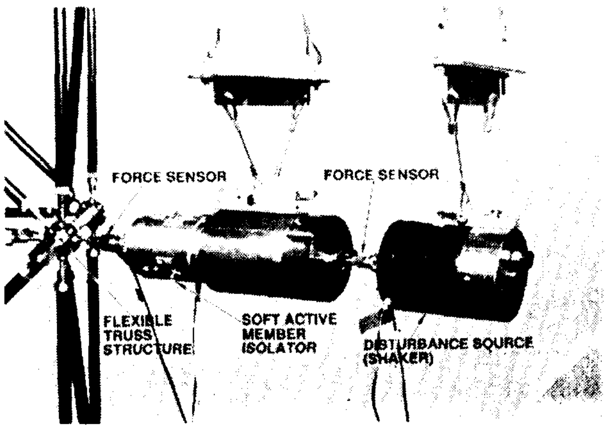


Figure 1 Vibration isolation experiment setup

Wc have implemented the method to isolate a disturbance source from a flexible aluminum truss structure. The experimental setup (Figure 1) simulates an elastic structure with a vibrating machine mounted on it, The isolator, placed between the structure and the vibration source, consists of a voice coil, a set of flexures and a 4 Kg mass. The stiffness of the flexure along with the mass determines the passive isolation break frequency which in our experiment is about 21 Hz. A load cell (performance sensor) installed between the isolator and the structure measures the force transmitted to the structure and provides the feedback signal in the active isolation control loop. The disturbance source is a proof-mass shaker suspended from the ceiling and attached to the isolation fixture via a stinger connector. The proof mass used in the experiment is approximately 2 Kg. System identification experiments on the structure (Figure 5) revealed the presence of approximately 14 lightly damped modes below 100 Hz.

LMS Algorithm

Figure 2 presents the continuous-time LMS algorithm framework, consistent with the treatment presented in [10]. Wc discuss convergence properties of the algorithm and some new results are presented for the case where the weights (w_i) are overparametrized.

We consider the disturbance signal $e_d(t)$ consists of a sum of sinusoids at different frequencies, amplitudes and phases. It is desired to cancel the disturbance signal $e_d(t)$ with the control signal $u(t)$ such that the net error signal $e = e_d - u$ is driven to zero. Note that the plant is assumed to be unity in the analysis. This is achieved by a plant inversion and is discussed in a later section, The error signal $e(t)$ is measured directly with a sensor, while the signal cd(t) is not available for measurement, It is assumed that cd(l) can be written as a linear combination of the elements of the known vector $x(t) \in RN$, *i.e.*, there is a constant vector $w^0 \bullet$ RN, such that,

$$e_d(t) = w^{0T} x(t) \qquad (1)$$

for all $t > 0$.

In practice, the components of the vector $x(t)$ are generated by filtering the measured reference input signal $\xi(t)$ which is correlated with $e_d(t)$. The existence of a constant vector w^0 in equation 1 requires that the components of vector $x(t)$ span cd(t) for all $t > 0$.

If w^0 is known, then the signaled can be canceled by the ideal signal u^0 constructed as,

$$u^0 = w^{0T} x(t) \qquad (2)$$

However, in practice the parameter vector w^0 is not known, and the ideal control signal (2) is replaced by the estimated quantity,

$$u = w^T(t) x(t) \qquad (3)$$

Here $w(t)$ is an estimate of w^0, which is tuned in real-time using the LMS algorithm,

$$w = \mu x(t) e(t) \qquad (4)$$

with adaptation gain $\mu > 0$. Let the tracking error be defined as,

$$e(t) = e_d(t) - u(t) \qquad (5)$$

and *the* parameter error be defined as,

$$\phi(t) = w^0 - w(t) \qquad (6)$$

Using equations 1,3,5 and 6, the tracking and the parameter errors can be related as follows,

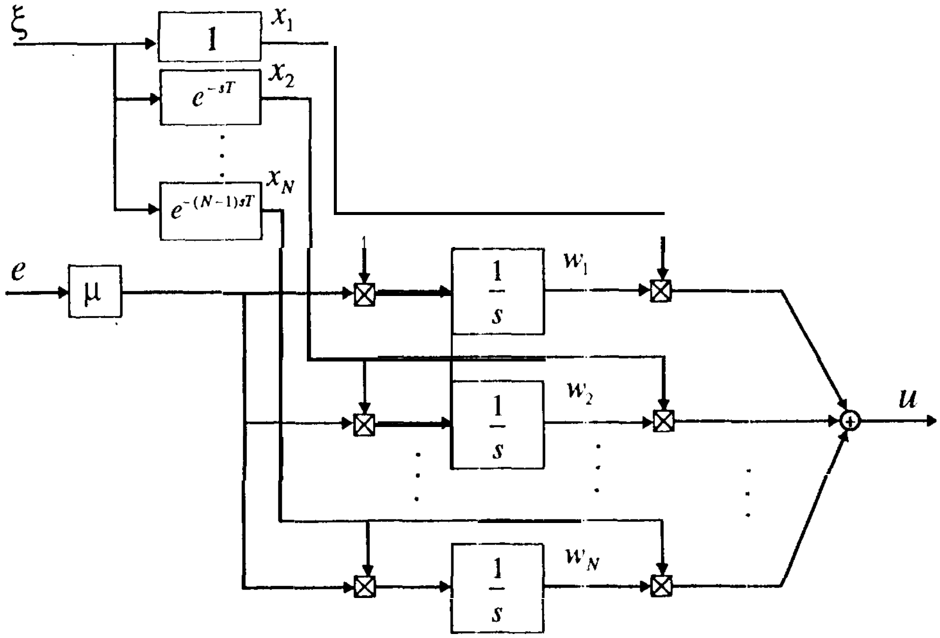


Figure 2 Continuous-time LMS algorithm [1O]

$$e(t) = \phi^T(t)x(t) \qquad (7)$$

assuming that the true parameter w^0 does not vary with time. The time derivative of equation 6 and the substitution of equations 4 and 7 yield,

$$\dot{\phi} = \dot{w}^0 - \dot{w} = -\mu xe \qquad (8)$$

The equation above characterizes the propagation of the parameter error.

Convergence Properties

We define the Lyapunov function,

$$V = \tfrac{1}{2}\phi^T\phi \qquad (9)$$

By taking the time derivative of the above equation and then rearran ging we get,

$$\dot{V} = -\mu e\phi^T x = -\mu e^2 \leq 0 \qquad (1o)$$

This proves that φ is bounded. Since $x(t)$ is bounded, $e(t)$ is also bounded due to equation 7. Furthermore, $e(t)$ approaches zero since $\dot{x}(t)$ is assumed to be bounded [10].

Reference [11] establishes the following results which indicates the rate of convergence.

If $x(t) \in RN$ is a bounded periodic vector function of $t \geq 0$, with period T_0, i.e.,

$$\|x(t)\| \leq \eta < \infty; \text{ for all } t \geq 0 \qquad (11)$$

$$x(t + 7.) = \dot{x}(t); \text{ for all } t \geq 0 \qquad (12)$$

and if there exists a w^0 such that (1) holds for all $t \geq ()$, and that the LMS algorithm is used to tune the parameter vector w using equations 3-8, then the error e in equation 7 approaches zero exponentially, i.e.,

$$|e(t)| \leq \eta e^{\alpha T_0} \|\phi(0)\| e^{-\alpha t} \qquad (13)$$

$$\alpha = \mu\lambda_{\min} \qquad (14)$$

$$M \equiv \frac{1}{T_0}\int_0^{T_0} x(t)x^T(t)dt \qquad (15)$$

where $\lambda_{\min}$ denotes the smallest positive eigenvalue of the symmetric non-negative definite matrix $M = MT \geq 0$. This result indicates that the rate of convergence of the error $e(t)$ is exponential and is determined by the smallest nonzero eigenvalue of the correlation matrix M. Note that the result is nonstandard in the sense that w can be overparametrized and M *can be* singular. The error converges exponentially fast as long as the correlation matrix M has at least one nonzero eigenvalue, and as long as there exists a parameter vector w^0 satisfying equation 1.

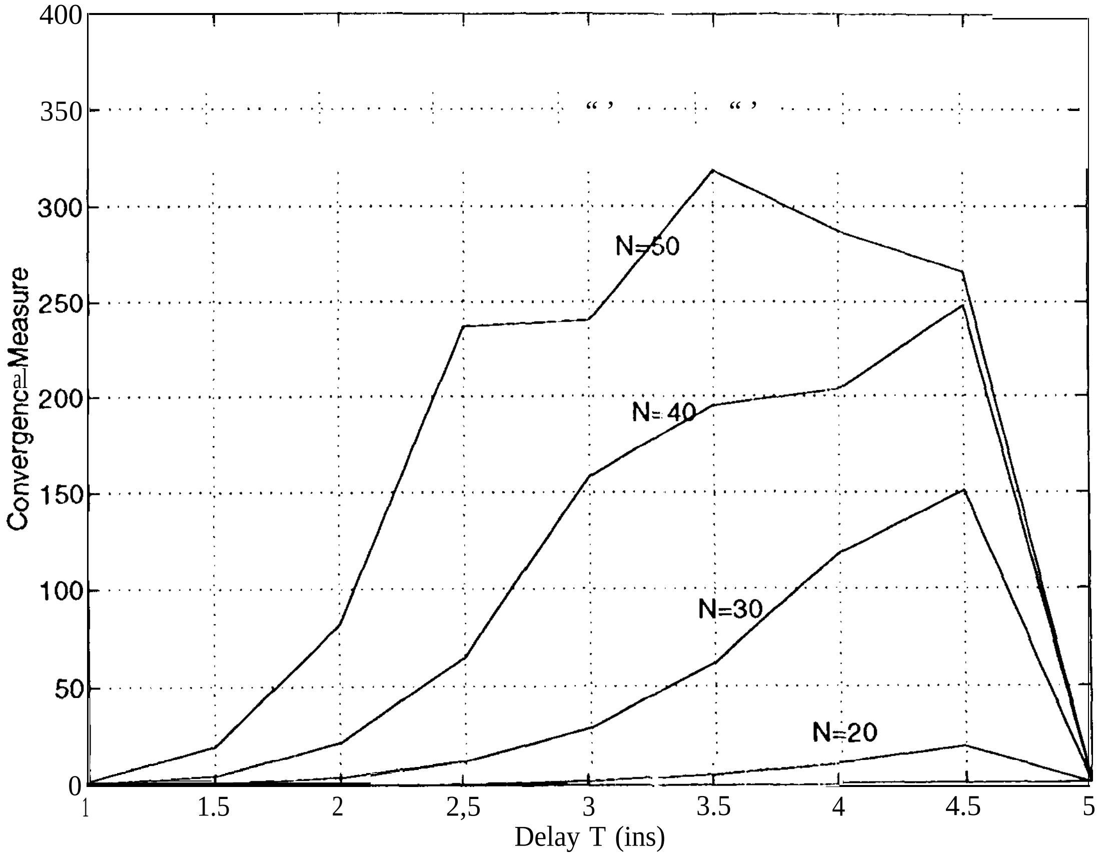


Figure 3 Convergence measure $S(N,T)$ for two tone problem with $\omega_2 = 2\omega_1$, over $\omega_1 \in [5, 50$ Hz], a.. a function of tap delay time 7' and number of taps N

In theory, the rate of convergence α in equation 14 can be made arbitrarily large by increasing μ. However this is not possible in practice since there will be limitations on the size of μ due to nonidealities such as the effect of the sampling delay which will be present when the scheme is implemented in a digital computer. To take into account the effect of sampling delay, we discretize the adaptation law (equation *4)* using Euler's method,

$$w_k = w_{k-1} + \bar{\mu} x_{k-1} e_{k-1} \tag{16}$$

where the discrete-time adaptation gain $\bar{\mu}$ is related to ttrc continuous-time gain μ by,

$$\bar{\mu} = T\mu \tag{17}$$

and 7' is the sampling period. It is well known from the literature [12] that a sufficient condition for convergence when using the discrete adaptation law (equation 16) is,

$$\bar{\mu} \leq \frac{2}{\lambda_{max}} \tag{18}$$

where λ_{max} denotes the largest eigenvalue of M. Hence the effect of sampling delay is to restrict the choice of μ to a quantity bounded by

$$\mu \leq \frac{2}{\lambda_{max}.T} \tag{19}$$

This indicates that a more realistic bound on the convergence rate is given by,

$$a = \mu\lambda_{min} \leq \frac{2\lambda_{min}}{\lambda_{max}.T} = \frac{2}{\kappa_1.T} \tag{20}$$

where the finite condition number κ_1 of M is defined as

$$\kappa_1 \equiv \frac{\lambda_{max}}{\lambda_{min}} \tag{21}$$

Equation 20 indicates that the rate of convergence of the error $e(t)$ to zero is increased as the finite condition number κ_1 or the sampling period 7' are reduced in size. Note that κ_1 is not independent of 7’ in our formulation as can be seen in the next section.

Two-tone Problem

in this section, the problem of adaptive vibration suppression is considered for a disturbance e_d which is composed of two tones at frequencies ω_1 and ω_2.

Let the measured reference signal $\xi(t)$ be given by the following sum of sinusoids,

$$\xi(t) = A_1 \sin(w_1 t + \varphi_1) + A_2 \sin(w_2 t + \varphi_2) \quad (22)$$

Define the filters F,(s) in terms of tap delay lines with a sequential delay of T

$$F_l(s) = e^{-(l-1)sT}, 1 = 1, \ldots, N \quad (23)$$

The components of $x = [.x_1, \ldots\ldots x_N]^T$ are obtained by filtering $\xi(t)$ through the inters in equation 23, i.e.,

$$x_l(s) = F_l(s)\xi(s), l = 1, \ldots, N \quad (24)$$

where $x(s)$ and $\xi(s)$ arc the Laplace transforms of $x(t)$ and $\xi(t)$ respectively. Then the correlation matrix M, as defined in equation 15, can be calculated in closed form as,

$$M(A_1,\omega_1 T, A_2, \omega_2 T, N) = \sum_{i=1}^{2} \frac{A_i^2}{2} * \begin{bmatrix} 1 & \cos\omega_i T & \cos 2\omega_i T & \cdots & \cos(N-1)\omega_i T \\ \cos\omega_i T & 1 & \cos\omega_i T & \cdots & \cos(N-2)\omega_i T \\ \cos 2\omega_i T & \cos\omega_i T & 1 & \cdots & \cos(N-3)\omega_i T \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \cos(N-1)\omega_i T & \cos(N-2)\omega_i T & \cos(N-3)\omega_i T & \cdots & 1 \end{bmatrix} \quad (25)$$

The notation $M(A_1, \omega_1 i“, A_2, \omega_2 T, N)$ emphasizes the dependence on the frequency ω_1, 01 $_2$ and amplitudes A_1, A_2 of each harmonic, as well as the tap-delay time 7’ and the number of taps N.

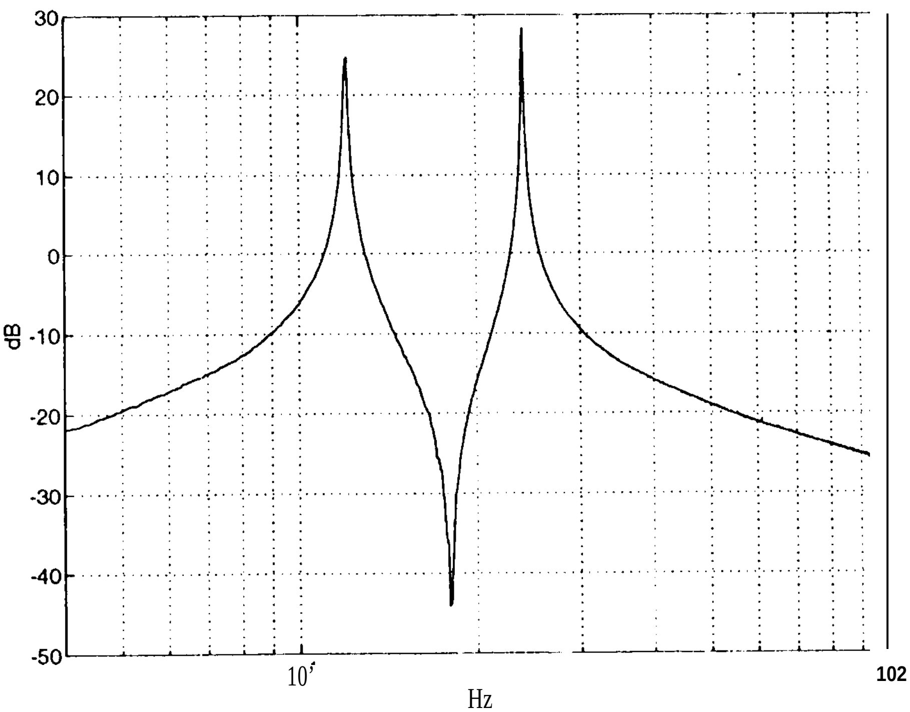
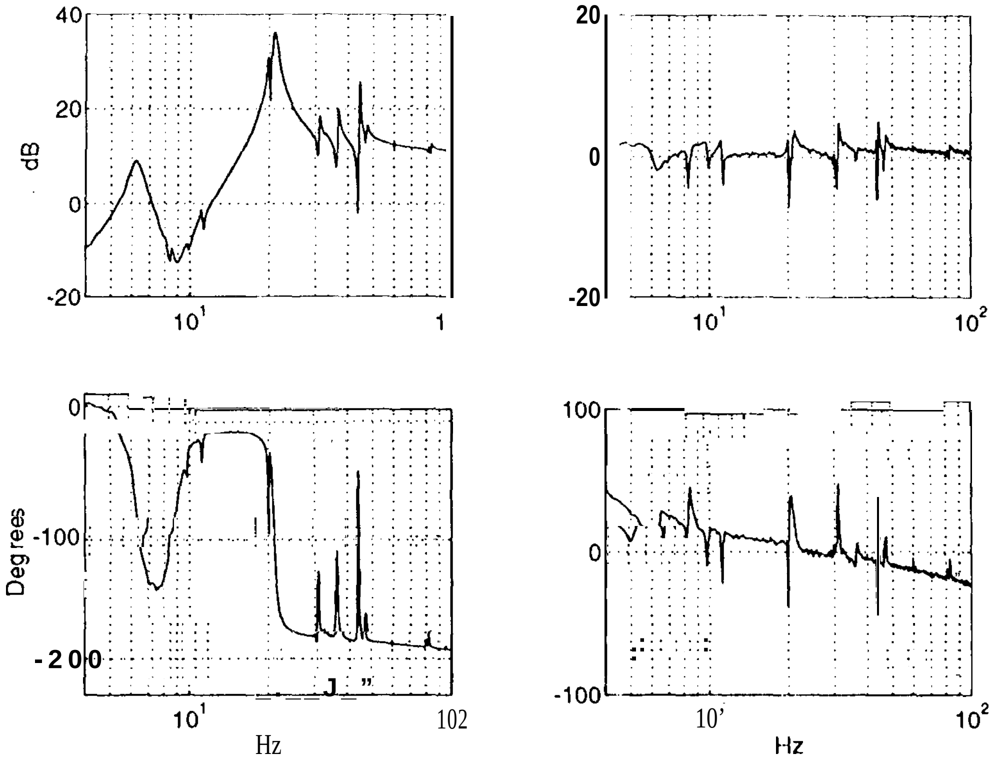


Figure 4 Experimentally realized LMS filter with damped integrator.



(a) (b)

Figure 5 Measured plant frequency response function (a) and plant cancellation (b).

If the disturbance frequencies ω_1 and ω_2 arc due to a single vibration source, they are often not completely arbitrary, but rather can be related in some deterministic manner, One common situation is when ω_2 is the second superharmonic of a fundamental at ω_1 i.e., the frequencies are related as

$$\omega_2 = 2\omega_1 \qquad (26)$$

By using the equation 26 and letting the amplitude of each harmonic component of $x(t)$ be unity (i.e., $A_1 = A_2 = 1$), the expression for M in equation 26 simplifies to,

$$M(A_1, \omega_1 T, A_2, \omega_2 T, N) = M(fT, N) \qquad (27)$$

where $f = \omega_1 / 2rr$. *Note* that the properties of M (i.e., κ_1) can conveniently be plotted against the dimensionless quantity fT and the number of taps N.

We used a simple graphical procedure for choosing T and N for our two lone harmonic disturbance experiment which is described next. Let the fundamental harmonic frequency ω_1 of the disturbance vary between a known bound,

$$\underline{\omega} \le \omega_1 \le \overline{\omega} \qquad (28)$$

Convergence results indicate that it is desirable to keep $1/(\kappa_1 7)$ as large as possible over the working bandwidth [Q,;]. We calculate the worst case value of $1/(\kappa_1 T)$ over the region $\underline{\omega} \le \omega_1 \le \overline{\omega}$ for various value of N and T. This defines the surface,

$$S(N, T) = \min_{\underline{\omega} \le \omega_1 \le \overline{\omega}} \frac{2}{\kappa_1 (M(fT, N))\ 7'} \qquad (29)$$

Note that the criteria (29) can be frequency weighted to incorporate prior knowledge concerning the location of the disturbance tone. The surface. equation *29*, is plotted in figure *3* as a function of N and T using the frequency range from 5 Hz ($\underline{\omega}$) to 50 Hz ($\overline{\omega}$), Good values for N and T for this case are easily seen as points where the surface is highest. While the convergence measure generally improves with larger number of taps, an arbitrarily small or large tap delay time will have an adverse effect on the convergence. For a small number of taps (below

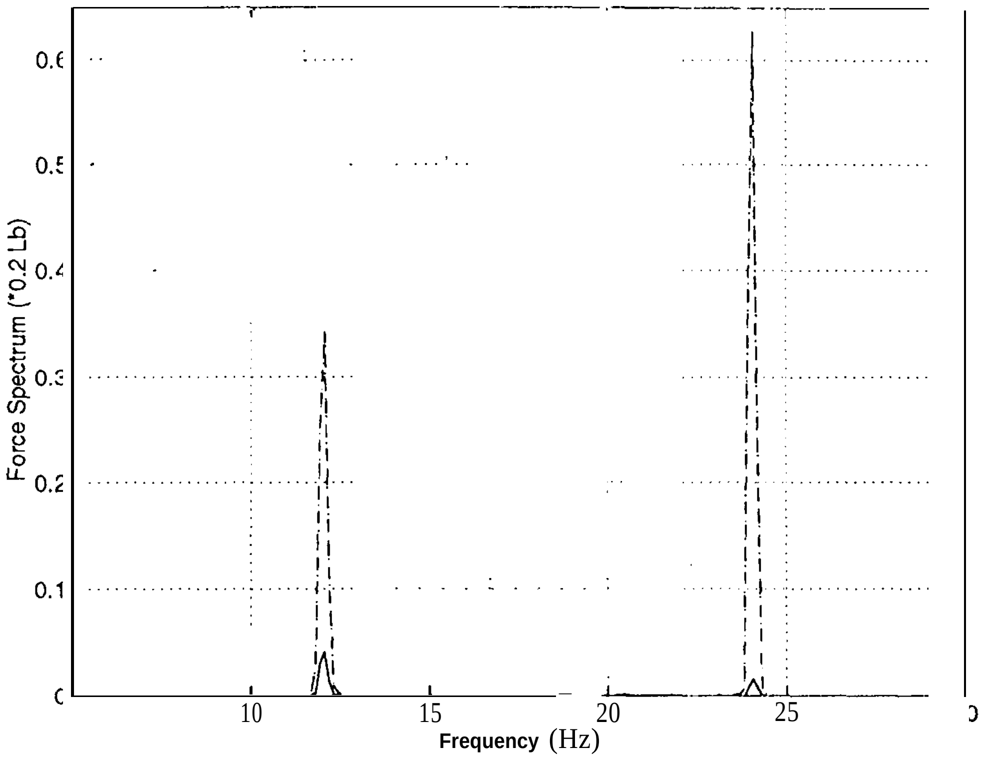


Figure 6 Measured closed loop (solid) and open loop (dashed) disturbance force spectrum for the two tone problem

30) or an arbitrarily small tap delay time (less than 2 milliseconds), the convergence measure is very small. Note that when the tap delay time is near 5 milliseconds, the convergence measure approaches zero since the rank of the correlation matrix approaches zero. This indicates that the disturbance component at $\omega_2 = 2\omega_1 = 100$ Hz (whose period is 10 milliseconds) can no longer be suppressed by the implementation,

If, as suggested in single tone cases, wc had chosen 1/4 of the wavelength of the highest frequency value of ω_2, the tap delay time would be 2.5 milliseconds. However as can be seen from Figure 3 that a more appropriate choice for the tap dctay time is 3.5 milliseconds with 50 taps. This was implemented in the experiment described next.

Experiment

The compensator, discretized at 2048 Hz, was implemented on a Heurikon HKV4F/33MHz 68040 processor running under VxWorks operating system. The controller had two components: 1) the proposed LMS algorithm and 2) an approximate plant inverse filter,

In the LMS algorithm, the number of taps (components of x) and the tap delay time (7) were 50 and 3,5 milliseconds respectively. There were 7 samples between each component of $x(t)$ and also between each component of $e(t)$, This required that both the storage buffers for $\xi(t)$ and e(t) be 350 (= 7*50) word long. Instead of shifting each element of each buffer one step backward (1400 read/write operations) for every cycle (of 1/2048th second), both $\xi(t)$ and e(r) were stored in two circular buffers to case the rest time implementation. This savings in real lime implementation came at a price of onc additional IF statement at every cycle and an additional buffer of 700 word long for storing a time index.

The control cycle was running at a faster rate than is required to implement a 3.5 millisecond tap delay time. This was to avoid aliasing of higher frequency components into the error signal e(t) and to maintain smoothness of the output control signal, 1 n a digital control implementation, it is important to

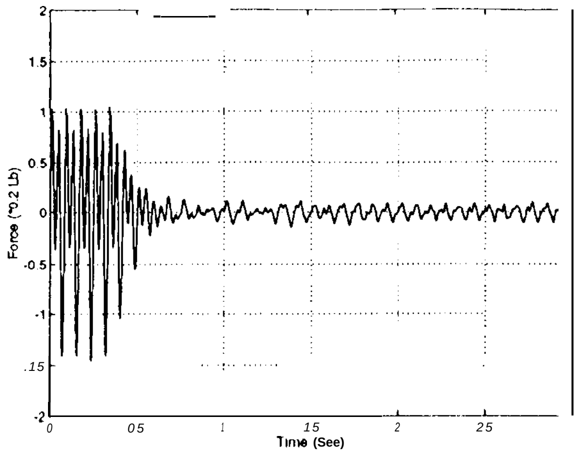


Figure 7 Measured time history as the loop is closed. Control starts at 300 ms.

run the update cycle much faster than the bandwidth of the actuator (voice coil) and the power amplifier combination.

The LMS filter response was measured using broad band disturbance input and is shown in Figure 4. The filter was implemented with damped integrators to limit the peak gains during this measurement. The response, which has two peaks at the frequencies (12 Hz and 24 Hz) of the reference input $\xi(t)$, corroborates Glover's results [9].

The plant was identified by minimizing the l_2 norm of the error [13]. An inverse filter was designed to keep the combined plant and the plant inverse function phase and magnitude variations within +/- 55° degrees and 5 dB respectively in the frequency band of 5 Hz to 100 Hz. This is necessary to maintain an overall phase margin of over 30° degrees for stability of the closed loop system since the disturbance frequency may vary between 5 Hz to 100 Hz. The plant response and the combined plant and plant inverse response are shown in Figure 5a and 5b respectively. This shows a phase variation of +/-55° degrees that guarantees a net phase margin of 35° degrees for stability.

Figure 6 and 7 show open and closed loop results when the disturbance input has frequency components at 12 Hz and 24 Hz. Figure 5 shows the frequency spectrum of the disturbance input before and after closing the loop. A combined reduction of the components of over 25 dB is achieved, Figure 6 shows a time record as the loop is closed. The controller is turned on at 300 millisecond. The peak value of the disturbance input is reduced from 0.3 Lb to less than 0.016 Lb in about 400 milliseconds which is longer than the predicted time. One reason for this is that we had to use a smaller $\bar{\mu}$ than the maximum allowed by equation 18 in order to keep the bandwidth of each peak of the filter very small, A part of this mismatch could also be attributed to the imperfect cancellation of the plant, digitization error, finite bit accuracy of the digital implementation, nonlinearities and other nonidealities.

Conclusion

An adaptive notch filter using the Least Mean Square algorithm was designed and implemented to isolate a multi-tone disturbance source. We showed that a proper choice of the number of taps and tap delay time can improve the convergence rate. An arbitrarily small or large delay time or a small number of taps will have an adverse effect on the convergence rate. Experimental implementation achieved an isolation performance level of 25 dB and convergence of the error to zero in 400 milliseconds. Our implementation required a plant inversion filter over the frequency range in which the frequencies of the disturbance varied. As a result the robustness of the implementation became dependent on the condition: that the plant dynamics does not change appreciably over time. In fact, for the method to be effective, sufficient modal damping should be present to accommodate small parameter variations. Future work will address the issue of increased performance and on-line plant identification. In addition, we will be directing our efforts towards applying the algorithm to a multi-axis isolation stage.

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